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# Theoretical study of the stress transfer in single fibre composites

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The stress transfer in single fibre reinforced composites is studied theoretically with the help of a finite difference type of approach. The results show that the critical fibre length for efficient stress transfer to the fibre is a unique function of the ratio between the Young elastic moduli of fibre and matrix. The effect of the bonding efficiency between the two components is studied and dramatic increases in the critical length are observed when adhesion falls below 30%. The results are compared to the predictions of other analytical and numerical approaches. A good quantitative agreement is found with available experimental data.

## 1. Introduction

Fibre reinforced materials are gaining increasing technological importance due to their great versatility and high performance. These materials often consist of discontinuous strong fibres embedded in a matrix, with the fibre axes oriented in the direction of the applied load [1, 2]. Since the elastic modulus of the fibre is typically much larger than that of the matrix, the axial elastic displacements of the two components can be very different. In order to rationalize the design of reinforced materials, it is thus of primary importance to have a detailed knowledge of the stress distribution induced by the applied load. Indeed, when discontinuous fibres are used, the attainment of good mechanical properties depends critically upon the efficiency of stress transfer between the matrix and the fibres. That efficiency is often characterized by the critical length, $l_c$, required of a fibre (of given diameter) to build up a maximum stress equal to that of an infinitely long fibre [3]. That length is closely related to the critical length determined by fibre pull-out tests [4].

Analytical equations for the variation of stress along discontinuous fibres in a cylindrically symmetrical model have been derived by Cox [5] and by Dow [6]. Both approaches, however, neglect the adhesion across the end face of the fibres and they fail to take into account local stress concentration effects near fibre ends. The importance of these assumptions has been demonstrated by finite element approaches [7–9]. Unfortunately, because of the large amount of computer time required by these approaches, the studies were restricted to very small systems and the importance of finite size effects could not be assessed.

The present paper attempts to tackle these problems using a finite difference type of approach. In that approach, the material is represented by a regular three-dimensional lattice whose nearest neighbour nodal points are linked by bonds having different elastic constants for the fibre and for the matrix. For a given external strain, these nodes are relaxed towards local mechanical equilibrium with their neighbours by a systematic sequence of operations which steadily reduce the net residual force acting on each node. In addition to a greater versatility, the major strength of the present approach over the finite element analysis is the many highly efficient algorithms available for the reduction of the residuals [10]. As a result, much larger systems can be investigated. The present work is mainly concerned with the study of a single fibre embedded in an infinitively large matrix. Finite size effects are studied and shown to play an important role in previous finite element analyses [7–9]. The effect, on the critical length, of the adhesion and of the difference in elastic moduli between fibre and matrix are investigated and the predictions of the model are compared to available experimental data. The extension of the present results to multi-fibre composites will be presented in a forthcoming publication.

## 2. Model

We start by describing our lattice model for a single fibre embedded in an infinite three-dimensional matrix. Fig. 1 gives a two-dimensional representation of the lattice in a $x$–$y$ plane passing through the centre of the fibre. The lattice is of the simple cubic type and comprises 300 nodes along the $y$-axis and 33 nodes in the transverse $x$- and $y$-directions. These dimensions were deemed sufficiently large in order to ensure that our results are size independent. Since the results turn out to be also independent of model details (see Section 3), the fibre diameter, $d$, was arbitrarily set equal to 3 lattice units. The elastic Young shear moduli of the matrix and of the fibre are denoted by $E_m$, $G_m$ and $E_f$, $G_f$, respectively.

The lattice is strained by a constant amount (1%) along the $y$-axis. This leads to displacements of the nodes along the different axes. In order to estimate these displacements, each node is relaxed towards local mechanical equilibrium with its neighbours by bringing to zero the net residual force acting on that node [11]. The liquidation of these residuals has



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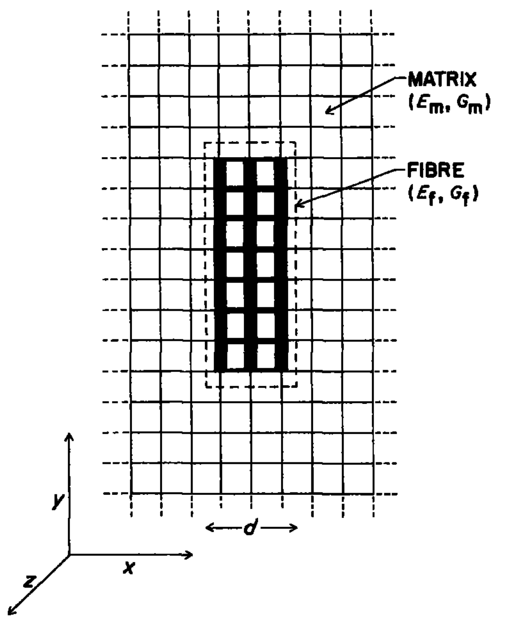


*Figure 1* Two-dimensional representation of the array of nodes in an $x$–$y$ plane passing through the centre of the fibre. The fibre diameter is set equal to $d$ = 3 lattice units. $E_m$,$G_m$ and $E_f$, $G_f$ are the Young and shear elastic moduli of the matrix and the fibre, respectively. The lattice is of the simple cubic type and comprises 300 nodes along the $y$-axis and 33 in the transverse $x$ and $y$ directions.

been performed using two well-known computational devices [10] which considerably speed up the convergence of the calculations. The first device, known as overrelaxation, consists not just in reducing the net residual for a node but, instead, deliberately goes further and gives to that residual a sighn that is opposite to that of the residuals for the neighbouring nodes. The second device is known as block relaxation and consists in relaxing more than one node at a time. The liquidation of the residuals was considered to be completed when the largest residual force for a node fell below a few per cent of the average force for a bond. The above relaxation process leads, for each node, to motions along the three coordinate axes. For simplicity, we assume these motions to be mutually independent and we focus on displacements along the $y$-axis [9]. Since the shear modulus of deformation predominates when it comes to load transfer between the matrix and the fibre, lateral motions in the $x$- and $z$-directions should be of secondary importance for the present study [8]. The validity of our assumption will also be discussed in Section 3.

A two-dimensional representation of the general deformation scheme of the model for a choice $E_f/E_m$ = 40 is shown in Fig. 2. Since the elastic modulus of the fibre is much larger than that of the matrix, the axial elastic displacements of the two components are very different. (The $y$-scale has been distorted so the effects can be easily seen.) The figure shows an important bending of radial lines at regions close to the fibre end. That observation is in sharp contrast to the assumption made in previous analytical approaches [6] that straight radial lines remain straight after composite deformation. The figure also indicates that, far away from the fibre end, radial lines recover their original direction perpendicular to the fibre axis, thus indicating that the limit of a very long fibre in an infinite matrix is attained.

## 3. Results and discussion

Fig. 3 shows the results for the tensile strain along a $y$-axis passing through the centre of the fibre. The strain is in units of the overall strain on the composite. Since the analysis is completely elastic, the value of the external strain has no real significance and stress concentration effects are independent of it. The figure is for different values of the ratio $E_f/E_m$, assuming perfect adhesion, i.e. no broken lattice bond at the interface between the matrix and the fibre. For simplicity, we

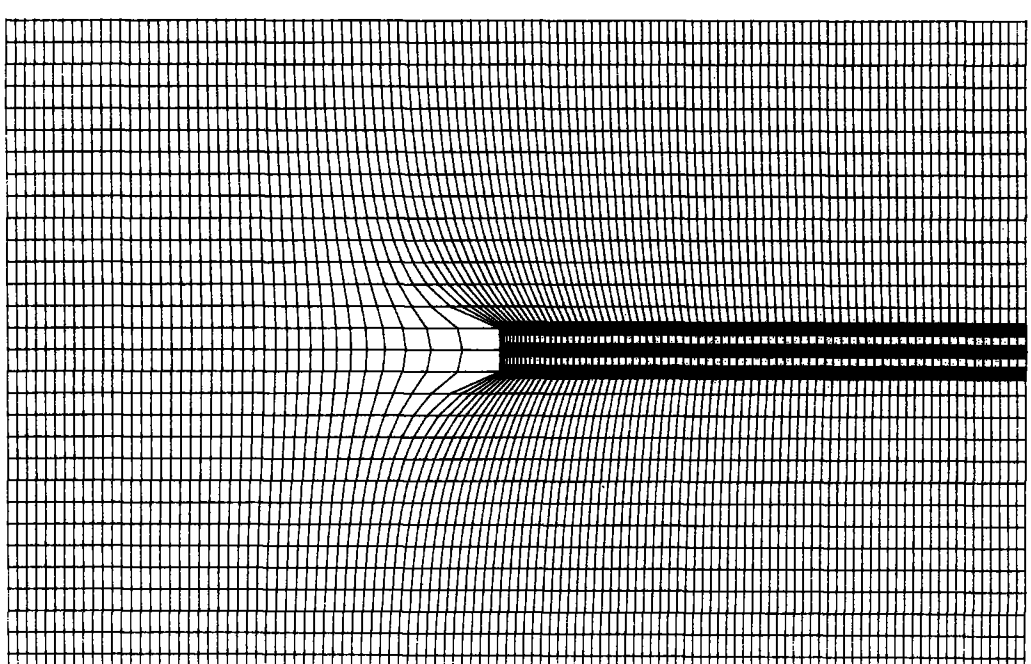

*Figure 2* General deformation scheme of the model on a $x$–$y$ plane, for a ratio $E_f/E_m$ = 40. The $y$-scale has been distorted so the effects can be easily seen. The fibre and the matrix were assumed isotropic.

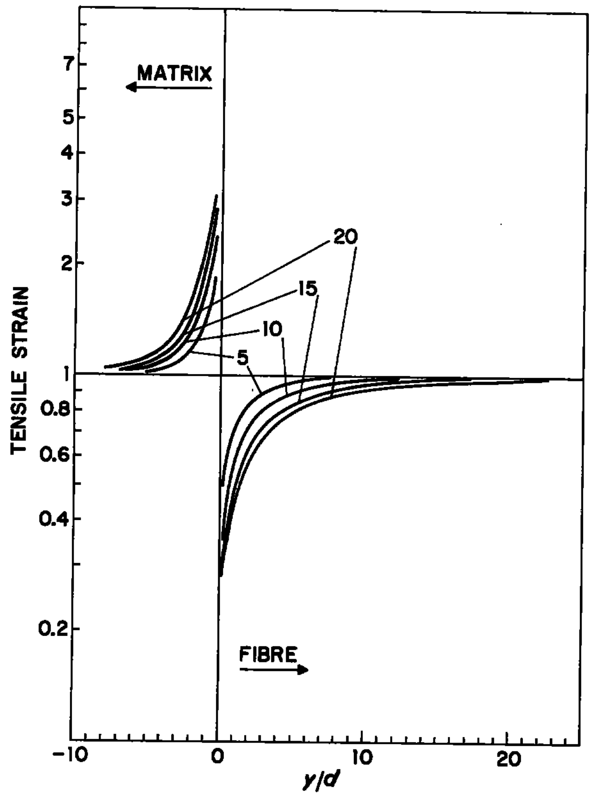


*Figure 3* Tensile strain near a fibre end, along a $y$-axis passing through the centre of the fibre. The strain is in units of the overall strain on the composite. The figure is for different values of the ratio $E_f/E_m$. We took $d = 3$ lattice units and assumed fibre and matrix to be isotropic.

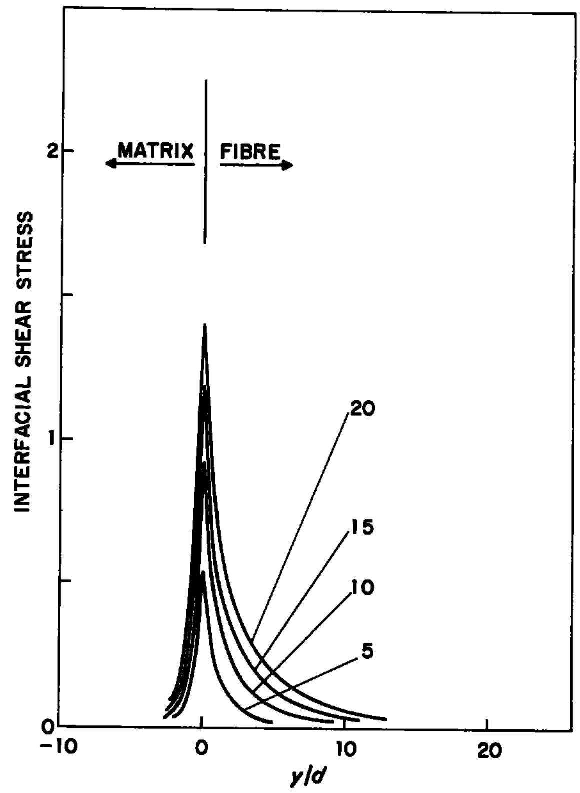


*Figure 4* Interfacial shear stress near a fibre end for the cases studied in Fig. 3. The stress is in units of the overall stress on the composite.

also assumed the matrix and the fibre to be isotropic, i.e. $G_m = E_m/2(1 + \nu)$, $G_f = E_f/2(1 + \nu)$ where we choose $\nu = 0.35$ for the Poisson ratio. The figure shows that the increase of axial strain in the fibre becomes less pronounced as the ratio $E_f/E_m$ is increased. The strain in the fibre does not start from zero because of the load transfer from the matrix across the fibre end. That transfer, neglected by Cox [5] and Dow [6] is seen to be quite important and accounts for more than 25% of the maximum fibre strain.

Calculations in which the fibre was assumed to be anisotropic, taking $G_f = E_m$, gave results identical to those of Fig. 3, to within a few per cent. We have also performed simulations using a cruder representation of the fibre by taking $d = 1$ lattice unit (instead of $d = 3$ units, see Fig. 1). Results, again, were similar to those of Fig. 3.

We now turn to a study of the importance of our neglect of the correlations between the nodes displacements in the different $x$-, $y$- and $z$-directions. If we assume that the deformation of the elementary volumes in Fig. 1 occurs at constant volume, we expect the motions of the nodes along the $y$-axis to be "hindered" by restoring forces exerted by the overall matrix in the $x$–$z$ plane. In order to estimate the importance of those effects, we turn to a mean-field type of approximation and express the restoring force acting on each bond as $F = E_m \Delta\varepsilon$ in which $\Delta\varepsilon$ is the difference between the overall strain on the composite and that on the bond being considered. Results obtained with that modified version of the model (for $E_f/E_m = 20, d = 1$) were found to be almost identical to those of Fig. 2, except for a slightly smoother variation of the tensile strain around the fibre end. The above calculations thus indicate that the build-up of strain in the fibre is quite independent of model details and is a unique function of the ratio between the elastic Young moduli of the fibre and the matrix.

The results for the interfacial shear stress corresponding to the cases studied in Fig. 3, are given in Fig. 4. The figure shows a sharp variation in shear stress within a distance equal to 2 to 4 fibre diameters from the fibre end. Not also that our curve for a ratio $E_f/E_m = 20$ is in good quantitative agreement with experimental data on the shear stress along a single Dural fibre in an Araldite matrix ($E_{Dural}/E_{Araldite} = 21$) [12].

Fig. 5 summarizes our results for the effect of $E_f/E_m$ on the critical length, $l_c$, of the fibre, i.e. the length necessary to build up in the fibre a maximum strain equal to 97% of that for an infinitely long fibre [2]. The knowledge of $l_c$ is of primary importance to ensure an efficient reinforcement effect by the fibre. The $l_c$ values are in units of the fibre diameter. Inspection of the figure shows that the critical length increases almost linearly with an increase in the fibre's Young modulus, $E_f$. That result is in sharp contrast to the prediction of previous analytical theories [4, 5] that $l_c \sim E_f^{1/2}$ [13]. Turning to comparison with experiment, Galiotis *et al.* [13] have recently measured the critical length of one polydiacetylene single crystal in an infinite epoxy matrix. The two components have a ratio $E_f/E_m = 16$, good bonding properties [14] and the critical length has been determined to be $l_c = 27$ fibre diameters [13]. These values are in good quantitative agreement with our results of Fig. 5, obtained

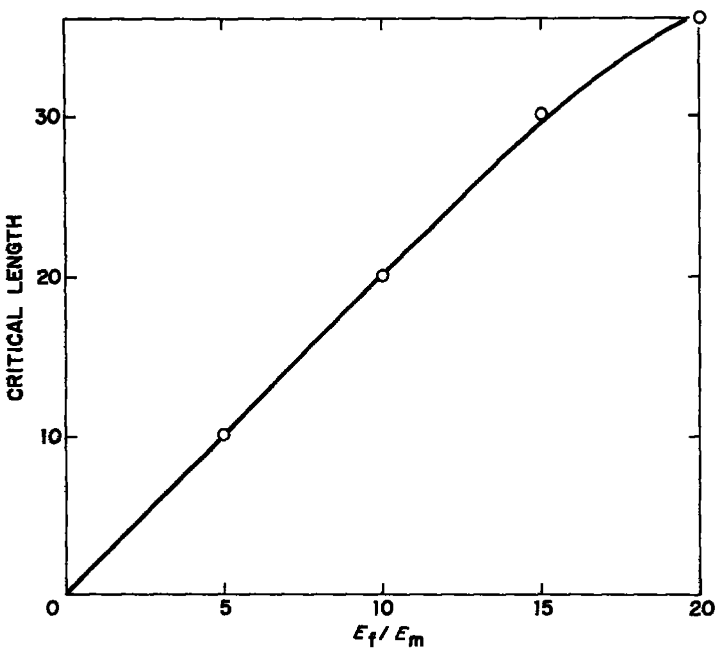


*Figure 5* Dependence of the critical length on the ratio $E_f/E_m$ for the cases studied in Fig. 3. The length is in units of the fibre diameter, $d$.

assuming perfect adhesion between fibre and matrix. Note, though, that the experimental data in [12] also show that the axial strain in the fibre at a distance $l$ far away from the ends ($l \gg l_c$) is lower than the overall strain on the composite, in contrast with our results in Fig. 3. That discrepancy, however, is due to a buckling effect of the fibre during the curing process [15].

The effect of the adhesion on the critical length is shown in Fig. 6, for a ratio $E_f/E_m = 15$ and $d = 1$ lattice unit. Changing the adhesion in the model was realized by breaking bonds at the fibre–matrix interface, with probability (1-adhesion factor). A decrease in the adhesion is seen to increase the critical length, that increase being particularly dramatic when adhesion becomes less than 30%. Alternatively, any experimental determination of the critical length in these composites could, with the help of Fig. 6, give useful estimates of the degrees of adhesion between fibre and matrix. Note, finally, that the results of Fig. 6 depend on the thickness of the interphase (thickness = fibre diameter in the figure). Any decrease of the interphase thickness will lower the adhesion values at which dramatic increases in $l_c$ are observed.

The results presented so far were for a single fibre embedded in a large cylindrical matrix of diameter up to 33 fibre diameters. The effect of a reduction in the matrix diameter on the build-up of strain in the fibre is displayed in Fig. 7, for $E_f/E_m = 15$. A decrease in the matrix diameter to 5 to $10d$ is seen to lead to an extremely slow build-up of strain and poor efficiency of load transfer between the matrix and the fibre. This result also clearly explains why the finite element simulations in [6], using a lattice diameter equal to $10d$, failed to reach the state of maximum strain in the middle of the fibre.

In conclusion, we have shown that the finite difference analysis developed in the present paper, can provide a useful tool for the study of the factors limiting the mechanical properties of fibre reinforced composites. In addition to a much greater versatility,

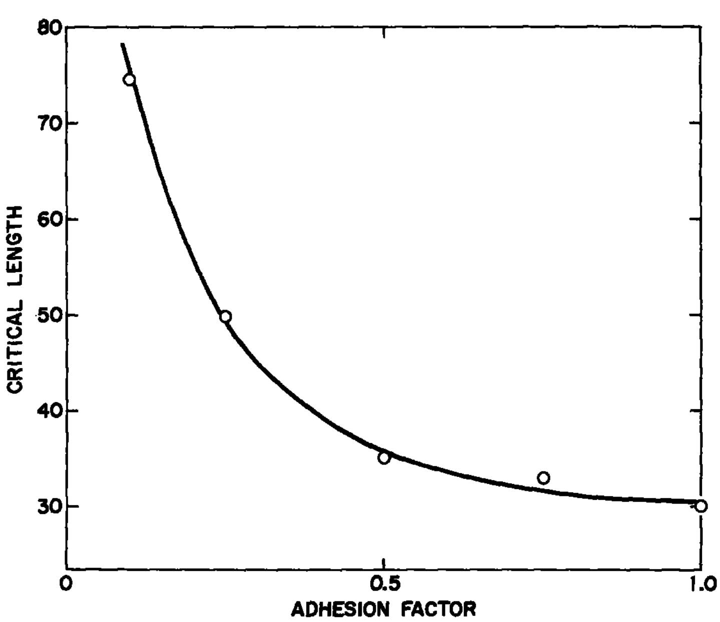


*Figure 6* Dependence of the critical length on the adhesion factor for a ratio $E_f/E_m = 15$. The figure is for $d = 1$ lattice unit.

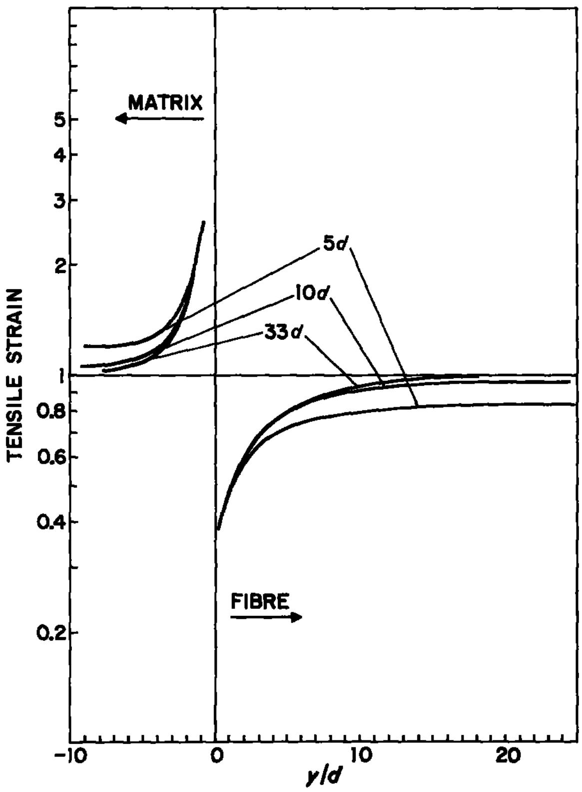


*Figure 7* Tensile strain near a fibre end, for different values of the matrix diameter. The figure is for $E_f/E_m = 15$, taking the fibre diameter $d = 1$ lattice unit.

the major strength of the present approach over the finite element analysis is the many highly efficient algorithms available for the reduction of the residuals. As a result, much larger systems can be studied and comparison to experiment becomes feasible. The present study dealt with the case of a single fibre composite. This is an essential step before multi-fibres composites can be considered, which will be the object of a forthcoming publication.

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